EXACT SOLUTION OF THE SHEAR EFFECT ON THE ANALYSIS OF LACED AND BATTENED BEAM-COLUMN

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Engineering Journal of the Univercity of Qatar; Vol. 18; 2005;
pp:105-120

Abstract

The complete derivations of the closed-form solution of the effect of both shear, bowing action and the fixed end action (F. E. A.) on stability are presented. Modified bowing functions including shear deformation and modified F. E. A. functions including shear effect using two methods are introduced. The first one is the total shear slope, which contains both the flexural and shear angles and the second one is the flexural shear slope which includes the shear angle directly with the flexural angle. A new 2-D TSM (tangent stiffness matrix) including shear effect is derived and a computational technique is set for this purpose. Numerical examples are solved and compared with other researchers. The validity and accuracy of the derived functions and TSM using method –1- is achieved while method –2- shifts away from its purpose.

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Key words: Axial shortening, Bowing, Large-deformation, Non-linearity, Open-webstructures, Shear deformation, Stability, Structural engineering.

I. INTRODUCTION

Briefly, the shear effect and the behavior of laced, and battened beam-columns (where shear effect dominates) were studied by many researches like, Williamson & Margolin, [3] Lin, F.J., and E.C. Glauser., [9] Renton, [4] A. Noor, [15] Noor & Weisstein,[5] AL-Sarraf, [10] Cheng & Pantelides, [1] Avent et al., [6] and many others.

For the conventional beam-column large-deflection theory, the most important studies, that paper depends upon are the works done by Saffan, [9] Oran & Kassimali, [12] and Kassimali, [13]; in addition to the main theory of exact Elastica, [16].

This paper adopts the equivalent shear parameter or the equivalent flexible web procedure first suggested by Niels & Newell(1943), and then adopted by Timoskenko & Gere, [2] Williamson & Margolin, [3] and Lin, [8].

II. THE EFFECT OF SHEAR ON NON-LINEARITY

The shear contribution in the entire deflection of a beam element as treated in the ordinary small deflection elastic theory is very simple; and, it is very small compared with the flexural deflection.

Actually there are two approaches developed in formulating the axial load effect when shear effect is included, [1], as shown in Fig. (1): -

2.1 The total shear angle effect (see Fig. 1-b).
2.2 The flexural shear angle effect (see Fig. 1-c).

In the first approach, the shear component of the axial load is assumed to act perpendicular to the tangent line of the total slope, which consists of the bending and shear slope. While in the second approach, the shear component of the axial load is acting perpendicular to the tangent line of the bending slope only. The differential equations for the two approaches are respectively as follows:-

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$$\frac{dy}{dx} = \psi + \beta = -\frac{dy_m}{dx} + \frac{nV_x}{AG} \quad \text{.....(1)}$$

$$\frac{dy}{dx} = \psi\left(1 + \frac{n}{AG}P\right) + \beta = -\frac{dy_m}{dx}\left(1 + \frac{n}{AG}P\right) + \frac{nV_x}{AG} \quad \text{.....(2)}$$

Where y_m is the deflection due to bending only.

Timoshenko and ***Gere***, [17], named the second approach the modified shear equation, and they introduced the critical load of the two approaches .

In the case of solid cross sections or columns of (I) sections the effect of shearing force can usually be neglected. While for built up columns consisting of struts connected by lacing bars or batten plates (i.e. the open web structures), the shear effect becomes of practical importance. It is considered dominant since the critical loads for these types of columns are always less than that for a solid column having the same area (A) and the same slenderness ratio (L/r). This decrease in the critical load is due primarily to the fact that the effect of shear on deflections is much greater for a laced column than for a solid bar. The actual value of the critical load depends upon the detailed arrangement and dimensions of the lacing bars, [17] .

So introducing a shear parameter or shear flexibility factor (u) – see Table 1 - enables any researchers to replace any section type (solid or lattice) by a fictitious equivalent solid section with the same shear parameter (u) values, [18]. This technique is called the equivalent shear parameter technique. Table (1) illustrates the governing equations of the shear parameter (u) assuming hinged ends. The typical types of the lattice columns and the sub symbols are shown in Fig. (2).

It is important herein to refer to the great works of ***Renton,*** [14], and ***Noor et al***. [11]. They concluded that ***Timoshenko and Gere***, [17]; formulas (the equations of Table -1-) are sufficient. Also ***Avent et al.*** [3], proved by the discrete field mechanics the validity of ***Timoshenko'***s equations.

The number of lacing panels has also a considerable effect on the (p_{cr}). Briefly speaking, if the number of panels is larger than (6), [17], then the assumption that the chords or legs are continuous and only hinges attach the diagonals is correct. The additional rigidity can be calculated from the assumption of continuity of the member so that the equation of the p_{cr} and then the shear parameters derived from it (i.e. Table 1) are always correct.

2.1 The shear effect on the flexural bowing

The mathematical expression of the flexural bowing effect is defined by:

$$\delta_b = \frac{1}{2}\int_0^{L_c}\left(\frac{dy}{dx}\right)^2 dx \quad \text{.....(3)}$$

The modified bowing formula ($\bar{C}_b$) and the modified bowing functions ($\bar{b}_1$) and ($\bar{b}_2$) including the shear effect represented by the shear parameter or the shear flexibility indicator (u) will be introduced for both methods (Meth.-1- and Meth.-2-).

Using the original definition of the bowing effect eq.(3) and setting up the elastic curve slope including the shear effect -dy/dx-, eq.(3) can be put into a new form, keeping in mined that ($\bar{dy}/dx$) is the slope of the elastic curve including the shear effect (for both methods)

The new modified shear-bowing formula can be introduced after long derivation introduced elsewhere[16]:-

$$\bar{C}_{b_1} = \bar{b}_1(\theta_1+\theta_2)^2 + \bar{b}_2(\theta_1-\theta_2)^2 \quad \text{.....(4)}$$

So, $\bar{b}_1$ (for Meth-1-) can be put in a general form, [16]: -

$$\bar{b}_1 = \frac{(\bar{C}_1+\bar{C}_2)^2}{4\pi^4\rho^2}\left[\frac{2\bar{\alpha}_1}{\sin^2 2\bar{\alpha}_1}(2\bar{\alpha}_1+\sin 2\bar{\alpha}_1\cos 2\bar{\alpha}_1)-2+\right.$$

$$\left.\frac{(\bar{\alpha}_1\cos 2\bar{\alpha}_1-\bar{\alpha}_1)(2\bar{\alpha}_1-\sin 2\bar{\alpha}_1)}{\sin^2 2\bar{\alpha}_1}\right] \quad \text{.....(5)}$$

Where:

$$\bar{\alpha}_1 = \frac{\pi}{2}\sqrt{\bar{\rho}_1} = \frac{\pi}{2}\left(\frac{\rho}{1-u\rho}\right)^{1/2} \qquad u\rho \neq 1,\ \rho = \frac{P}{P_e} = \frac{PL^2}{EI\pi^2} \text{ (positive value)}$$

Then $\bar{b}_2$ (for Meth.-1-) can be put into a general form, [16]:-

$$\bar{b}_2 = \frac{\left(\bar{C}_1 + \bar{C}_2\right)^2}{8\pi^4\rho^2}\left[\frac{\left(2\bar{\alpha}_1 - 2\bar{\alpha}_1\cos 2\bar{\alpha}_1\right)\left(2\bar{\alpha}_1 - \sin 2\bar{\alpha}_1\right)}{\sin^2 2\bar{\alpha}_1}\right] \quad \text{.....(6)}$$

Again $\bar{\alpha}_1 = \frac{\pi}{2}\sqrt{\bar{\rho}_1} = \frac{\pi}{2}\left(\frac{\rho}{1-u\rho}\right)^{1/2}, u\rho \neq 1$

$\rho = \frac{P}{P_e} = \frac{PL^2}{EI\pi^2}$ (positive value) and "u" is the shear parameter

In the tensile axial force and using the same procedure, the modified bowing function, for Meth.-1- is:-

$$\bar{b}_1 = \frac{\left(\bar{C}_1 + \bar{C}_2\right)^2}{4\pi^4\rho^2}\left[\frac{2\bar{\alpha}_1\left(2\bar{\alpha}_1 + \sinh 2\bar{\alpha}_1\cosh 2\bar{\alpha}_1\right)}{\sinh^2 2\bar{\alpha}_1}\right] - 2 +$$

$$\frac{\left(\bar{\alpha}_1\cosh 2\bar{\alpha}_1 - \bar{\alpha}_1\right)\left(2\bar{\alpha}_1 - \sinh 2\bar{\alpha}_1\right)}{\sinh^2 2\bar{\alpha}_1} \quad \text{.....(7)}$$

$$\bar{b}_2 = \frac{\bar{C}_1 + \bar{C}_2}{8\pi^4\rho^2}\left[\frac{\left(2\bar{\alpha}_1 - 2\bar{\alpha}_1\cosh 2\bar{\alpha}_1\right)\left(2\bar{\alpha}_1 - \sinh 2\bar{\alpha}_1\right)}{\sinh^2 2\bar{\alpha}_1}\right] \quad \text{.....(8)}$$

Where: $\bar{\alpha}_1 = \frac{\pi}{2}\sqrt{\bar{\rho}_1} = \frac{\pi}{2}\left(\frac{\rho}{1+u\rho}\right)^{1/2}$

$\rho = \frac{P}{P_e} = \frac{PL^2}{EI\pi^2}$ ρ (positive value) and (u) is shear parameter.

By a long way of squaring, Integrating and simplifying of eq.(3) one has the final form of the modified shear-bowing formula of Meth.-2-

$$\bar{C}_{b_2} = \frac{\omega^2 L}{4P^2}\csc^2\omega L\left(M_1^2 + 2M_1 + M_2\cos\omega L + M_2^2\right) +$$

$$\frac{\omega}{4P^2}\cot\omega L\left(M_1^2 + 2M_1M_2\cos\omega L + M_2^2\right) - \frac{\left(M_1 + M_2\right)^2}{2P^2L} \quad \text{.....(9)}$$

Where: $\omega = \frac{\pi}{L}\sqrt{\bar{\rho}_2} = \frac{2\bar{\alpha}_2}{L} = \frac{\pi}{2}\sqrt{\bar{\rho}_2}$ (for Meth.-2-)

And by the same procedure used in Meth.-1-, we can introduce the modified shear bowing functions for Meth.-2-:-

For compressive axial force($\rho > 0$)

$$\bar{b}_1 = \frac{\left(\bar{C}_1 + \bar{C}_2\right)^2}{4\pi^4\rho^2}\left[\frac{2\bar{\alpha}_2\left(2\bar{\alpha}_2 + \sin 2\bar{\alpha}_2\cos 2\bar{\alpha}_2\right)}{\sin^2 2\bar{\alpha}_2} - 2 +\right.$$

$$\left.\frac{\left(\bar{\alpha}_2\cos 2\bar{\alpha}_2 - \bar{\alpha}_2\right)\left(2\bar{\alpha}_2 - \sin\bar{\alpha}_2\right)}{\sin^2 2\bar{\alpha}_2}\right] \quad \text{.....(10)}$$

$$\bar{b}_2 = \frac{\left(\bar{C}_1 + \bar{C}_2\right)^2}{8\pi^4\rho^2}\left[\frac{\left(2\bar{\alpha}_2 - 2\bar{\alpha}_2\cos 2\bar{\alpha}_2\right)\left(2\bar{\alpha}_2 - \sin 2\bar{\alpha}_2\right)}{\sin^2 2\bar{\alpha}_2}\right] \quad \text{.....(11)}$$

Where: $\bar{\alpha}_2 = \frac{\pi}{2}\sqrt{\bar{\rho}_2} = \frac{\pi}{2}\left(\rho\left(1 + u\rho\right)\right)^{1/2}$: $\rho = \frac{P}{P_e} = \frac{PL^2}{\pi^2 EI}$ (positive value)

For tensile axial force ($\rho<0$)

$$\bar{b}_1 = \frac{\left(\bar{C}_1 + \bar{C}_2\right)^2}{4\pi^4\rho^2}\left[\frac{2\bar{\alpha}_2\left(2\bar{\alpha}_2 + \sinh 2\bar{\alpha}_2\cosh 2\bar{\alpha}_2\right)}{\sinh^2 2\bar{\alpha}_2} - 2 +\right.$$

$$\left.\frac{\left(\bar{\alpha}_2\cosh 2\bar{\alpha}_2 - \bar{\alpha}_2\right)\left(2\bar{\alpha}_2 - \sinh 2\bar{\alpha}_2\right)}{\sinh^2 2\bar{\alpha}_2}\right] \quad \text{.....(12)}$$

$$\bar{b}_2 = \frac{\left(\bar{C} + \bar{C}_2\right)}{8\pi^4\rho^2}\left[\frac{\left(2\bar{\alpha}_2 - 2\bar{\alpha}_2\cosh 2\bar{\alpha}_2\right)\left(2\bar{\alpha}_2 - \sinh 2\bar{\alpha}\right)}{\sinh^2 2\bar{\alpha}_2}\right] \quad \text{.....(13)}$$

Where: $\bar{\alpha}_2 = \frac{\pi}{2}\sqrt{\bar{\rho}_2} = \frac{\pi}{2}\left(\rho\left(1 - u\rho\right)\right)^{1/2}$: $\rho = \frac{P}{P_e} = \frac{PL^2}{\pi^2 EI}$ (positive value)

The bowing functions ($\bar{b}_1$) and ($\bar{b}_2$) for both methods with u=0.1 is shown in Fig. (3). In order to check the validity, accuracy and convergence of the bowing function derived a special program ***CHECK*** is prepared, and the new modified bowing functions including shear deformation are valid in all the values of their domains[16].

3-The Effect of Transverse Loading on the Non-Linearity (Fixed End Action -F.E.A.)

It is obvious that the effect of transverse loading has a significant effect on predicting and evaluating the critical load, and the entire stability non linearity of the element. This effect is what is known as the fixed end action.

$$Mf_1 = -F\frac{y}{\theta_a} \quad \text{.....(14)}$$

Where "F" is the concentrated load at any distance (x) along the span, and (y) is the deflection of the elastic curve. After long derivation detailed elsewhere, [16], the following FEA functions for both methods are introduced: -

$$\mathrm{Mf_0} = \frac{\mathrm{wL^2}}{8} \times \frac{(1+0.0016\mathrm{u})}{(0.002)} \approx \mathrm{wL^2}\,\frac{(1+0.0016\mathrm{u})}{0.016} \qquad \text{for Meth.-1-} \quad \text{.....(15)}$$

$$\mathrm{Mf_0} = \frac{\mathrm{wL^2}}{8} \times \left(\frac{2\mathrm{u}}{1.24\left[-1+(1+0.064\mathrm{u})^{1/2}\right]} \right) \approx \frac{\mathrm{wL^2}}{5} \times \frac{\mathrm{u}}{\left[-1+(1+0.064\mathrm{u})^{1/2}\right]} \qquad \text{For Meth.-2-} \quad \text{.....(16)}$$

These functions are illustrated in Fig. (4) for different values of (u), where the values for Meth.-2- are completely equal to ***Lin***'s [9], values.

4- The Modified TSM Including Shear Effect

Using the same procedure used by ***Saffan ,*** [15], the TSM of the 2-D beam-column element in the relative deformations is:-

$$t_{ij} = \frac{\partial s_i}{\partial u_j} + \frac{\partial s_i}{\partial \rho}\frac{\partial \rho}{\partial u_j} \quad \text{for (i, j = 1 to 3)} \quad \text{.....(17)}$$

Where "S" are the nodal forces:-

$$S_1 = M_1 = \frac{EI}{L}\left(\overline{C}_1\theta_1 + \overline{C}_2\theta_2\right) \quad : \quad S_2 = M_2 = \frac{EI}{L}\left(\overline{C}_2\theta_1 + \overline{C}_1\theta_2\right)$$

$$S_3 = Q = EA\left(\frac{u}{L} - \overline{C}_b\right),\ \overline{C}_b = \overline{b}_1\left(\theta_1+\theta_2\right)^2 + \overline{b}_2\left(\theta_1+\theta_2\right)^2 \quad \text{.....(18)}$$

All the values of (t_{ij}) must be derived, [16]:

$$t_{11} = \frac{\partial S_1}{\partial u_1} + \frac{\partial S_1}{\partial \rho}\frac{\partial \rho}{\partial u_1} \quad \text{.....(19)}$$

$$\frac{\partial S_1}{\partial u_1} = \frac{\partial S_1}{\partial \theta_1} = \frac{EI}{L}\overline{C}_1 \quad \text{.....(20)}$$

$$\frac{\partial S_1}{\partial \rho} = \frac{EI}{L}\left(\overline{C}_1'\overline{\theta}_3 + \overline{C}_2'\theta\right) \quad \text{.....(21)}$$

Then the final TSM form is, [16]:

$$t = \frac{EI}{L}\begin{bmatrix} \overline{C}_1 + \dfrac{\overline{G}_1^2}{\pi^2 H} & \overline{C}_2 + \dfrac{\overline{G}_1\overline{G}_2}{\pi^2 H} & \dfrac{\overline{G}_1}{H} \\ \overline{C}_2 + \dfrac{\overline{G}_1\overline{G}_2}{\pi^2 H} & \overline{C}_1 + \dfrac{\overline{G}_2^2}{\pi^2 H} & \dfrac{\overline{G}_2}{H} \\ \dfrac{\overline{G}_1}{H} & \dfrac{\overline{G}_2}{H} & \dfrac{\pi^2}{H} \end{bmatrix} \quad \text{.....(22)}$$

$$\begin{aligned} \overline{G}_1 &= \overline{C}_1'\theta_1 + \overline{C}_2'\theta_2 \\ \overline{G}_2 &= \overline{C}_2'\theta_1 + \overline{C}_1'\theta_2 \end{aligned} \quad \text{.....(23)}$$

$$H = \frac{\pi^2}{\lambda^2} + \left[\overline{b}_1'\left(\theta_1+\theta_2\right)^2 + \overline{b}_2'\left(\theta_1-\theta_2\right)^2\right] \quad \text{.....(24)}$$

In his paper, ***Al-Sarraf,*** [1]**,** presented modified stability functions ($\overline{C}_1$) and ($\overline{C}_2$) including shear effect by using the total shear slope (Meth.-1-). He also

presented the modified stability functions including shear effect by using the flexural shear slope (Meth.-2-) in a paper not published yet. The graphs of these functions for u=0.1 is shown in Fig. (5).

Using the same computational technique used in ***Oran,*** [12], ***Oran & Kassimali,*** [13], or ***Kassimali,*** [7], in all what the system equilibrium equation, the calculation of member axial force, and the steps of the algorithm concern, a computer program can be established.

5- Numerical Examples

Three different examples using different "u" values are analyzed using the no shear effect (Meth. -0-), the total shear effect (Meth. -1-) and the flexural shear effect (Meth. -2-), by means of both S.D. and L.D. procedures.

5.1 Case study no. 1(cantilever beam)

The elliptical integral exact solution of this problem is made by ***Bisshopp*** & ***Drucker*** (1945) by assuming that the beam is perfectly inextensible, a relatively large area is assumed in the present analysis in order to approximate a similar condition of inextensibility. Table (2.a) summarizes the properties of this problem where its obvious that the cantilever is analyzed using one beam-column element, its also obvious that the effect of shear in this problem is equal to zero since the value of the shear parameter "u" reaches zero value.

Many researchers studied this problem where no instability occurred.. Table (2.b) summarizes the efforts of many investigators as a comparison of P-Δ results. Also Fig. (6) illustrates the P-Δ behavior of this case study by analyzing it by putting u=0 which represents (Meth.-0-) and the shear parameter value u=3.553E-5 to be considered as shear effect analysis which represents (Meth.-1-&Meth.-2-) both in S.D. non-linear analysis (no bowing and no change in length) which is denoted in dashed

lines, and in L.D. analysis denoted in solid lines. Both horizontal and vertical displacements of point "B" are drawn.

It can be seen that all Meth.-0-, Meth.-1-, and Meth.-2-are in excellent agreement with the exact solution, this is because of the very small value of "u" which means no significant role of shear in this solid section problem. ***Langrangian*** and ***Eulurian*** iterative solutions of ***Oran*** and ***Kassimali***[13] (1976) have also a good agreement with the exact solution as it can be noticed from Table (2.b) while ignoring the bowing effect shifted the results away by about **11.325%.**

From Fig. (9) it can be noticed that the shear effect in both S.D. and L.D. gives no difference in horizontal and vertical deflections, the final displacements occur at **1.78E-3 kN** have **-5.84%** and **0.12%** difference between S.D. and L.D. analysis for vertical and horizontal deflections respectively.

5.2 Case study no.2 (Simply-supported slender beam)

This simple case study was solved by ***Kinney,*** [8], in (1957), by using the small deflection linear unit-load method technique to evaluate the shear and flexural deflections separately. He found that the shear to flexural deflection ratios for all the four cases of slenderness ratio **(L/D)** as detailed in table (3.b)

In the non-linear analysis both, it is not practical nor actual behavior to separate the pure shear deflection from the pure flexural deflection because they both act simultaneously.

Table (3.a) summarizes the properties and the load condition of this problem and Fig. (7) and table (3.b) show the results. It is clear that the beam is loaded in its middle span by a concentrated load (P) and that the four cases of different L/D ratios are taken to illustrate the effect of slenderness ratio on the

value of shear parameter and accordingly the shear effect on the (P- Δ) behavior. By observing table (3.b) and Fig. (7), it can be seen that when L/D decreases or "u" increases, the divergence between Meth.-0-(no shear included), Meth.-1- (total shear angle method) and Meth.-2- (flexural shear angle) increases .

5.3 Case no. 3 (Truss cantilever)

Although one of the most important conditions of the equivalent shear parameter method is that the panels must not be less than 5-6 in order to give an accurate "**u**" value calculated according to table (1). It might be a good task to take this case study to illustrate the shear effect in lattice sections.

As it was previously indicated in this paper that the lattice, trussed or open-web structures exhibit a relatively higher shear deformations than the solid and closed-web structures. Using the present formulations an equivalent member with solid cross sectional areas and shear parameter "u" is to replace the actual member. Its equivalent area, inertia and length must be detailed according to the approach described earlier. In order to explain this, the case study of truss cantilever previously solved by other researchers like ***Noor*** [10], is an absolutely perfect example to study. The actual and equivalent properties are detailed in tables (5.a and b) respectively, and their sketches are in Fig. (8.a), notice that "u" becomes more noticeable (**0.2506**) in comparison with the cases studied so far for the single members modeling represented by the present work.

it can be noticed that (Meth.-1-/ Meth.-0-) at P= 53.4 kN for the L.D**.** analysis gives an increase of **7.615%** while Meth.-2- exhibits **6.63%** increment which is purely the effect of shear deflection on the load-deflection behavior and indeed it means that both Meth.-1- and Meth.-2- introduce an enormous difference in the load-deflection behavior.

At the same load stage ***Noor***`s compared method gives a decrease of **8.02%** in the deflection value with Meth.-0- (no shear effect) analysis if we use "0.5E". Using "E" the results are very close to ***Noor***'s. This is because his method did not included the geometrical non-linearity effect of L.D. very important parameters which are the stability and the flexural bowing effects which are included in Meth.-0-, although he included the important change in length parameter. Meanwhile Meth.-1- and Meth.-2-give a **16.22%** increment and **15.17%** increment respectively over ***Noor's*** deflection result.

6- Conclusions

The following conclusions are stated: -

1. Shear effect plays a major role governing the P-Δ behavior of the open-web structure.
2. Meth. -1- exhibits an excellent response in the cases solved regardless of the "u" value, while Meth. -2- failed in giving a logical response in the cases of high values of "u" i.e., the open-web structures, meanwhile it gives excellent results when "u" approximately < 1.
3. The equivalent shear parameter technique represents one of the most important and successfully applied procedure that can deal with analyzing all types of structures including the most complex structures that consist

of large number of elements. This technique is efficient since it idealizes at least five lattice panels by one equivalent element.

4. For the beam-column case studies, the solution gives an excellent agreement of the present approach with exact large Elastica solved problems.
5. Changing the type of bracing system in the equivalent elements with the number of elements kept constant leads to a misleading result in the P-Δ behavior of the open-web structure.
6. The higher the "E" values the less deflection of the structure.

Appendix I: References

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Appendix II: Notations

The following symbols are used in this paper

a, b, α, A_d, A_e A_l, A_h, I_d, I_l, I_h	Laced section properties
$A_{f,}$ A_w	Flange and web areas
$C_{1,}$ C_2	First and second stability function for no shear effect
C_b	Bowing factor for no shear effect
E, G	Elastic and shear modulus
I, J	Second and Polar moment of inertia of section
L, L_c	Original and updated lengths
M	End moment
M_{f0}	Fixed End Action momets
n	Shape factor
P, P_{cr}	Axial force, Critical axial load
P_e	***Euler*** buckling load for pin-pin column = $EI\frac{\pi^2}{L^2}$
r	Radius of gyration
u	Shear parameter
V	Shear force
$\bar{C}_1$, $\bar{C}_2$	First and second stability function for shear effect
$\bar{b}_1$, $\bar{b}_2$	First and second bowing function for shear effect
$\bar{C}_b$	Bowing factor including shear effect
$\bar{\rho}$	Axial force parameter including shear effect
$\bar{y}$	Deflection in 2-D analysis including shear effect
θ	Slope of the elastic curve

ψ	Flexural slope = $\frac{dy_m}{dx}$
β	Shear slope
ν	Poisson's ratio
ρ	Axial force parameter for no shear effect
δ_b	Axial shortening due to flexural bowing

The following abbreviations are used in this paper

F. E. A.	Fixed end action.
L. D.	Large deflection analysis.
Meth. –0-	The no shear effect approach.
Meth –1-	The total shear slope approach.
Meth –2-	The flexural shear slope approach.
S. D.	Small deflection analysis.
T. S. M.	Tangent stiffness matrix.

List of Figures

Fig.1. : (a) Shear contribution of flexural curvature.

(b) First approach.

(c) Second appraoch.

Fig.2. : Types of lacing.

Fig.3. : P/Pe versus bowing function curves including shear effect by both methods (u=0.1) .

Fig.4. : P/Pe versus fixed-end moment including shear effect for various shear parameter (u) using both methods (U.D.L. over full length).

Fig.5. : P/Pe versus stability function curves including shear effect by both methods (u=0.1).

Fig.6 : Load-deflection curve for cantilever beam.

Fig.7 : Load-Deflection Curve for the Simply Supported Slender Beam.

Fig.8-a : Actual and Equivalent elements of truss cantilever.

Fig.8-b : Load-Deflection curve for the truss cantilever.

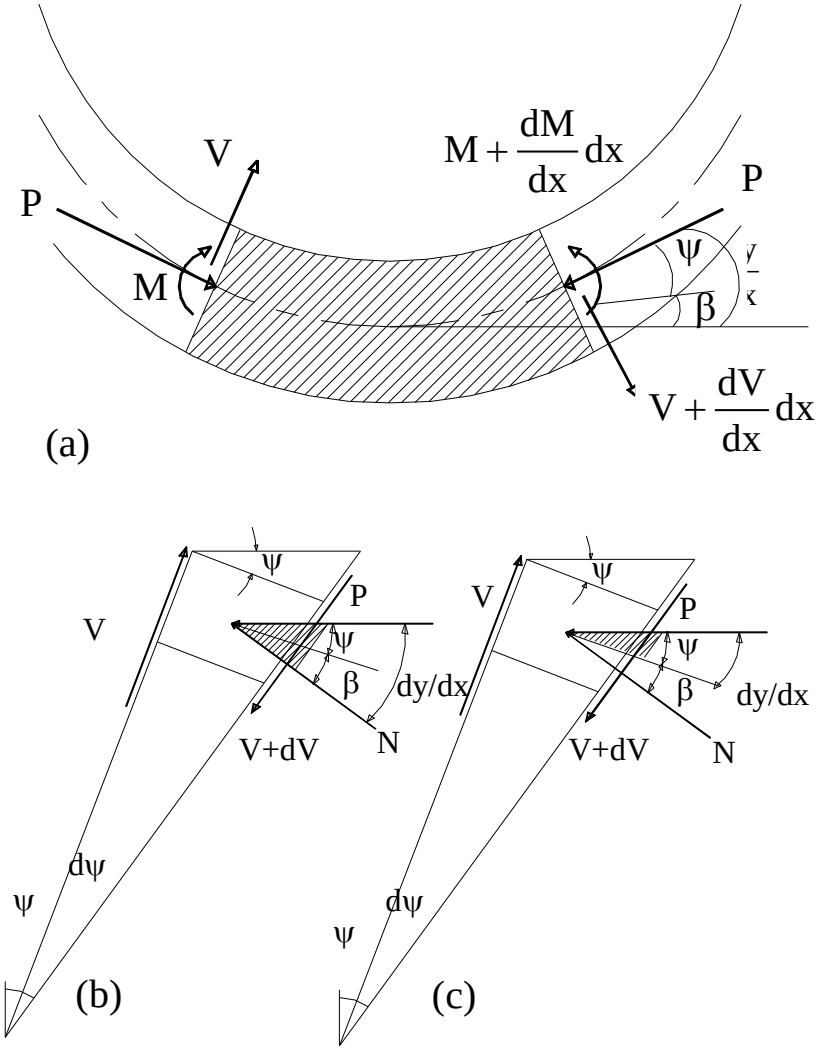

Fig. 1. (a) Shear contribution of flexural curvature; (b) First approach; (c) Second approach.

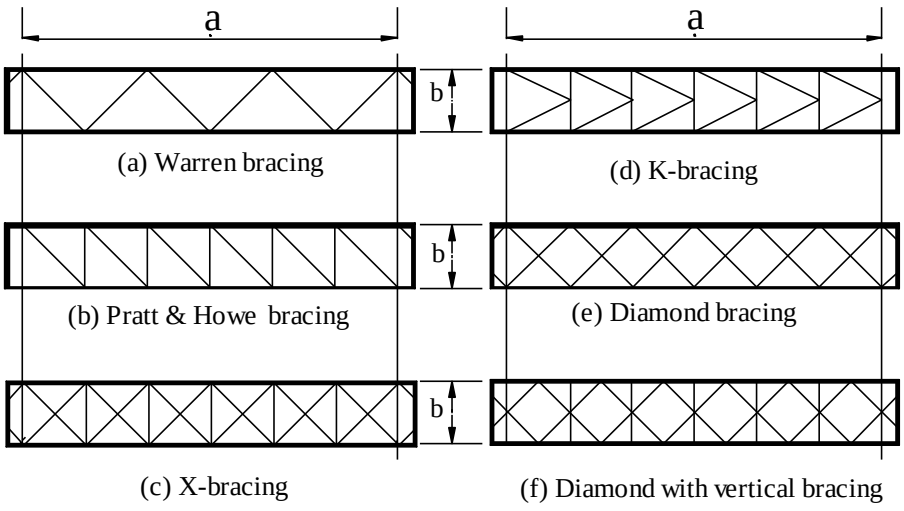

Fig. 2. Types of lacing

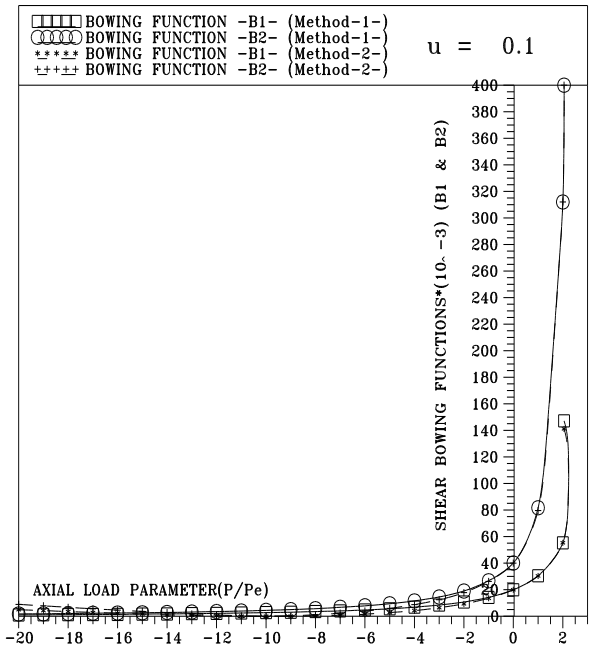
BOWING FUNCTION -B1- (Method-1-)
BOWING FUNCTION -B2- (Method-1-)
BOWING FUNCTION -B1- (Method-2-)
BOWING FUNCTION -B2- (Method-2-)
u = 0.1
SHEAR BOWING FUNCTIONS*(10.-3) (B1 & B2)
AXIAL LOAD PARAMETER(P/Pe)

Fig. 3. P/P_e versus bowing function curves including shear effect by both methods (u=0.1)

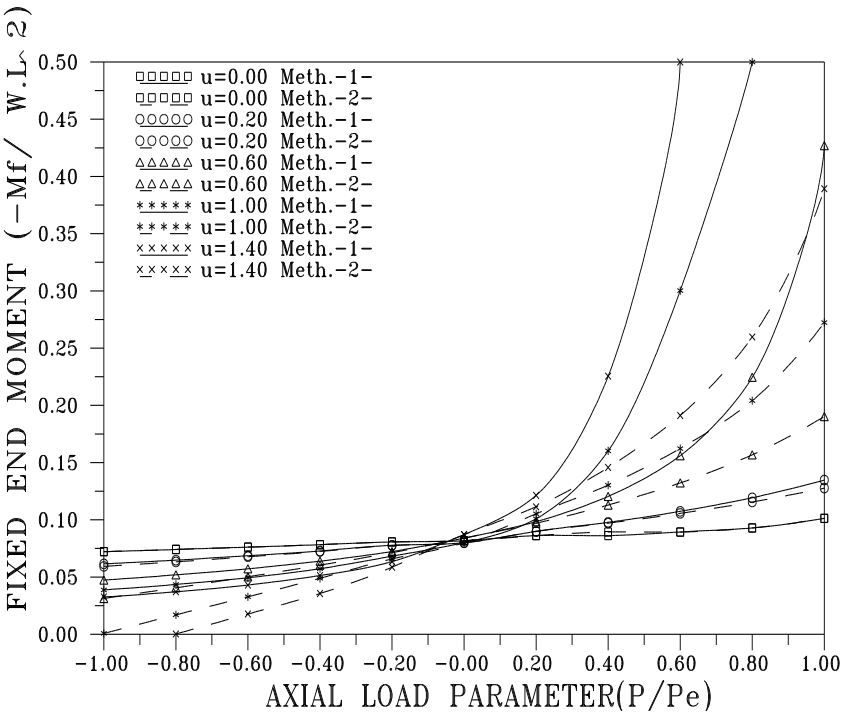

Fig. 4. P/P_e versus fixed-end moment including shear effect for various shear parameter (u) using both methods (U.D.L. over full length)

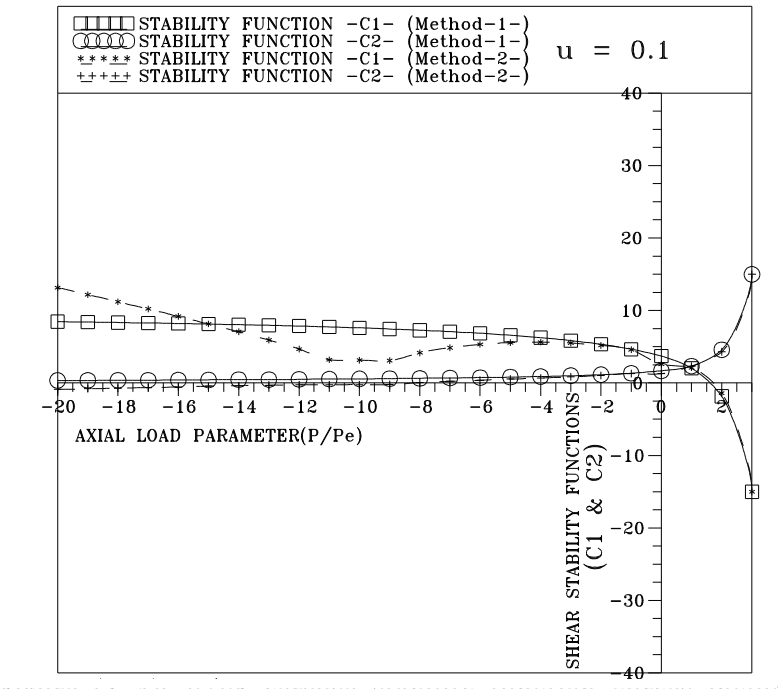
STABILITY FUNCTION -C1- (Method-1-)
STABILITY FUNCTION -C2- (Method-1-)
STABILITY FUNCTION -C1- (Method-2-)
STABILITY FUNCTION -C2- (Method-2-)
u = 0.1
AXIAL LOAD PARAMETER(P/Pe)
SHEAR STABILITY FUNCTIONS (C1 & C2)

Fig. 5. P/P_e versus stability function curves including shear effect by both methods (u=0.1)

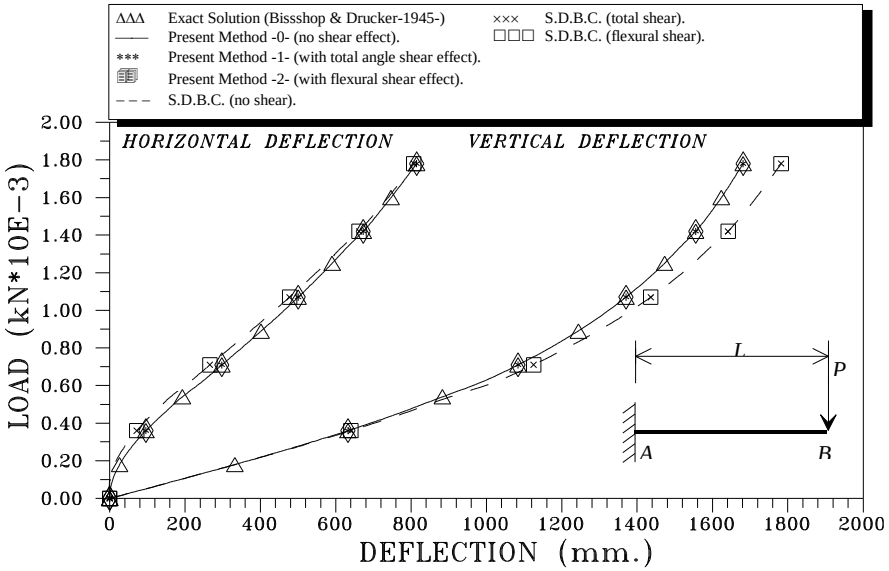

Fig. 6. Load-Deflection Curve For cantilever Beam

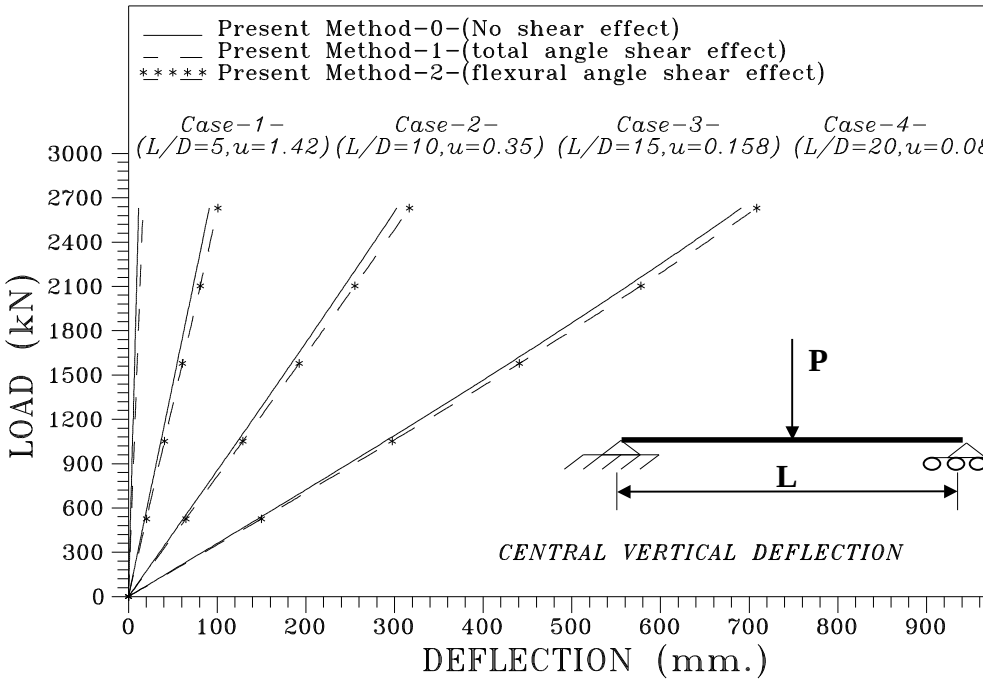

Fig. 7. Load-Deflection Curve for the Simply Supported Slender Beam

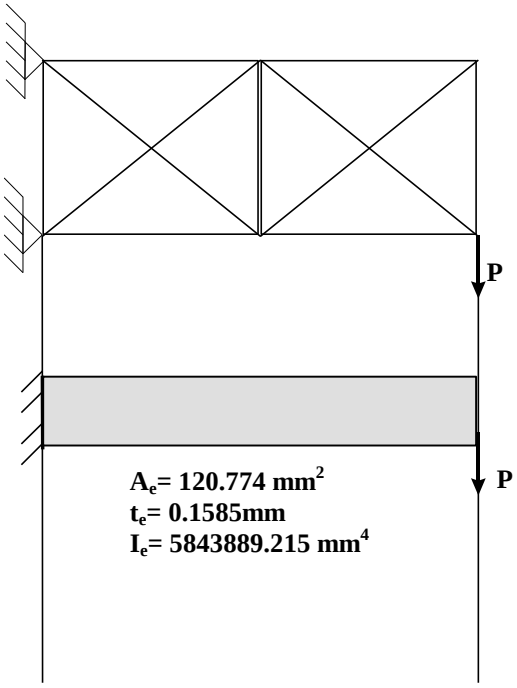

Fig. 8-a. Actual and Equivalent elements of truss cantilever

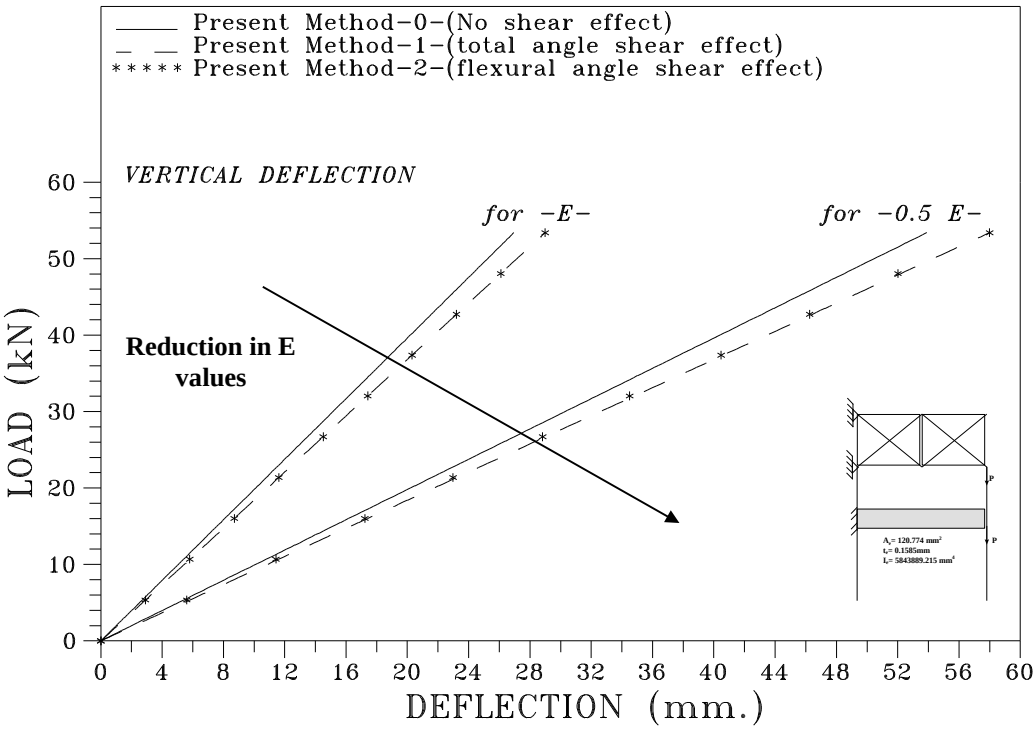

Fig. 8-b. Load-Deflection curve for the truss cantilever

Table 1 The shear parameter (u) governing equations

No.	*Type of Section*	*Shear Parameter* **(u)**
1	Solid Cross Section	$nP_e/AG = \frac{2n(1+\nu)\pi^2}{(L/r)^2}$
2	Pratt-Bracing	$\frac{EI\pi^2}{L^2}\left(\frac{1}{A_d E.\sin\alpha\cos^2\alpha}+\frac{b}{a.A_h.E}\right)$
3	Howe-Bracing	$\frac{EI\pi^2}{L^2}\left(\frac{1}{A_d.E.\sin\alpha\cos^2\alpha}+\frac{b}{a.A_h.E}\right)$
4	X- Bracing	$\frac{EI\pi^2}{L^2}\left(\frac{1}{A_d.E.\sin\alpha\cos^2\alpha}\right)$
5	Warren-Bracing	$\frac{EI\pi^2}{L^2}\left(\frac{1}{A_d.E.\sin\alpha\cos^2\alpha}\right)$
6	Vierendeel or Batten Lacing	$\frac{EI\pi^2}{L^2}\left(\frac{a.b}{12.E.I_h}+\frac{a^2}{24.E.I_l}+\frac{n.a}{b.A_h.G}\right)$

Table 2.a : Cantilever Beam Properties

Elm. "(i)"	*Length "L_i (mm)"*	*Area "A_i (mm^2)"*	*Elastic Modulus "E_i (Mpa)"*	*Moment of Inertia "I_i (mm^4)"*	*Shear Prmtr. "u_i"*	*Joint Load "P12 (kN)"*
1	2540	64516	68.95	416231.43	3.553*10-5	1.78*10-3

Table 2.b : Compression of Solutions For Cantilever Beam

No.	*Method*	*Type of Analysis*	*No. of Element*	*No. of Increment*	*Vertical Deflection (mm)*
1	Elastica	Exact Solution	1		1682.55
2	Linear	Beam-Column S.D.	1	20	3444.24
3	Oran & Kassimali (1976)	Beam-Col.L.D. Incremental Eularian (with no bowing)	1	20	2004.82
4	Oran & Kassimali (1976)	Beam-Column L.D. Incremental Eularian	1	20	1777.77
5	Oran & Kassimali (1976)	Beam-Column L.D. Iterative Lagrangian	1	20	1725.19
6	Oran & Kassimali (1976)	Beam-Column L.D. Iterative Eularian	1	20	1682.55
7	Present Method "0"	Beam-Column L.D. "No Shear"	1	20	1682.55
8	Present Method "1"	Beam-Column L.D. "with Shear"	1	20	1682.55
9	Present Method "2"	Beam-Column L.D. "with Shear"	1	20	1682.55

Table 3.a : Properties of Simply-Supported Slender Beam

Case	*Element Section "(i)"*	*Length L_i(mm.)*	*Depth of Section D_i(mm.)*	*L/D*	*Area Ai(mm^2)*	*Moment of Inertia "I_i(mm^4)"*	*Section Modulus S_i(mm^3)*	*Elastic Modulus E_i(Mpa)*	*Shape Factor "n"*	*Shear Paramet. "u_i"*	*0.6Fy "Mpa"*	*P_{all} "Mpa"*	*Concent. Load "kN" at Mid-span*
1	12W27	1524	12.00	5	4896.7644	84911210.8	560437.6	200000	2.89	1.419	148.93	5093.47	2630.00
2	12W27	3048	12.00	10	4896.7644	84911210.8	560437.6	200000	2.89	0.3548	148.93	5093.47	2630.00
3	12W27	4572	12.00	15	4896.7644	84911210.8	560437.6	200000	2.89	0.1577	148.93	5093.47	2630.00
4	12W27	6096	12.00	20	4896.7644	84911210.8	560437.6	200000	2.89	0.0887	148.93	5093.47	2630.00

Table3.b : Comparison of Simply-Supported Slender Beam Results

		Deflections (mm.)			*Max. Deflection Ratio "Shear/Flexure"*				
Case	*No. of Increments*	*Present Meth.-0-*	*Present Meth.-1-*	*Present Meth.-2-*	*$(\delta_v/\delta_b)_1$*		*$(\delta_v/\delta_b)_1$*		*Kinney (1957)*
					L.D.	*S.D.*	*L.D.*	*S.D.*	*S.D.*
1	10	13.829	14.327	14.164	0.4308	0.4314	D.N.W.	0.4261	0.60
2	10	86.314	86.624	86.478	0.1072	0.1078	0.1065	0.1064	0.15
3	10	196.543	196.567	196.552	0.0468	0.0479	0.0463	0.0475	0.07
4	10	311.854	311.859	311.856	0.02518	0.02698	0.02526	0.0266	0.04

Table 4.a : Truss Cantilever Actual Properties

Element "(i)"	*Length "L_i(mm)"*	*Area "A_i(mm^2)"*	*Moment of Inertia "I_i(mm^4)"*	*Elastic Modulus "E_i(Mpa)"*	*Shape Factor "n_i"*
Horizontal	762	161.29	83.25	68950	1.2
Vertical	1016	129.03	69.37	68950	1.2
Diagonal	1270	129.03	69.37	68950	1.2

Table 4.b : Truss Cantilever Equivalent Properties

Type of Bracing	*Bracing Angle "α"*	*Equivalent. Length "L_i(mm)"*	*Diagonal Area "Ad(mm^2)"*	*Equivalent Thickness "t_e(mm)"*	*Equivalent Area "A_e(mm^2)"*	*Equivalent Inertia "I_e(mm^4)"*	*Shear Parameter "u_i"*	*"u" Govern. Equivalent*	*Joint Load* (kN)
X-Bracing	36.869	1524	129.032	0.1585	120.774	5843889.215	0.2506	Eq. 3-7	-

Table 4.c : Comparison of Solutions For Truss Cantilever

No.	*Method*	*Type of Analysis*	*No. of Element.*	*No. of Increment*	*Vertical Deflection at "E"(mm.)*
1	Richard & Goldberg(1963)	Beam Theory Linear S.D.	10	12	20.955
2	A.Noor (1974)	Beam Theory L.D. (G.N.L.)	10	12	25.005
3	A.Noor (1974)	Beam Theory L.D. (G&M.N.L.)	10	12	49.89
4	Present Method "0"	Beam-Column L.D.	2	12	26.95 -E- 53.89 -0.5E-
5	Present Method "1"	Beam-Column L.D. "with Shear"	2	12	28.994 -E- 57.98 -0.5E-
6	Present Method "2"	Beam-Column L.D. "with Shear"	2	-	28.73 -E- 57.46 -0.5E-